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Title: Spherical Aberrations Final Focus and Solenoid Scans

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Spherical Aberrations
Final Focus and Solenoid Scans

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6-3-20

Motivation

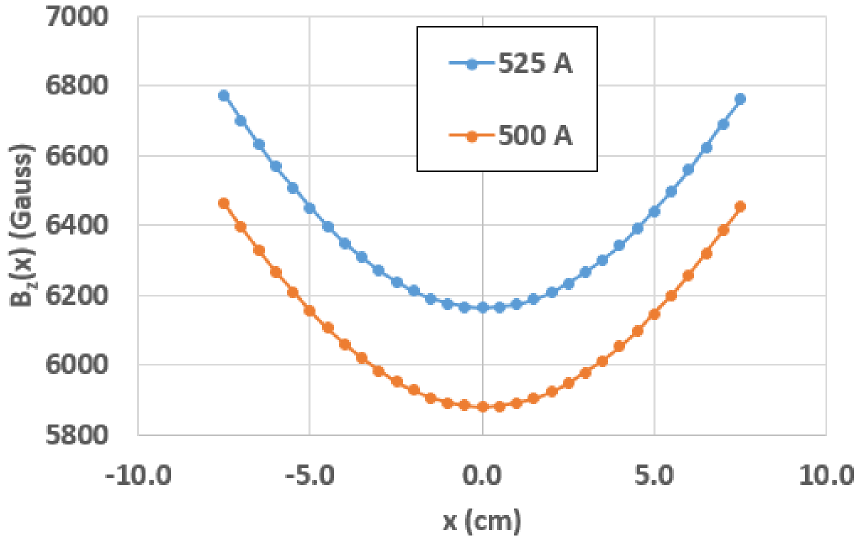
- Part of larger effort to quantify all contributions to spot size on target
  - Low dose pulse formats
    - FWHM for all four pulses is ~ 0.6 mm (based on best data)
  - High dose pulse formats
    - FWHM is ~0.6 mm for P1 and P2
    - Best FWHM for P3 and P4 is ~ 1.0 mm
- Contributions to spot size on target
  - Beam emittance
  - Chromatic aberrations due to the finite energy spread of the beam
  - Spherical aberrations due to the final focus solenoid
  - Beam motion (kicker smear, BBU, etc)
  - Beam target dynamics
  - Finite target thickness

What are spherical aberrations?

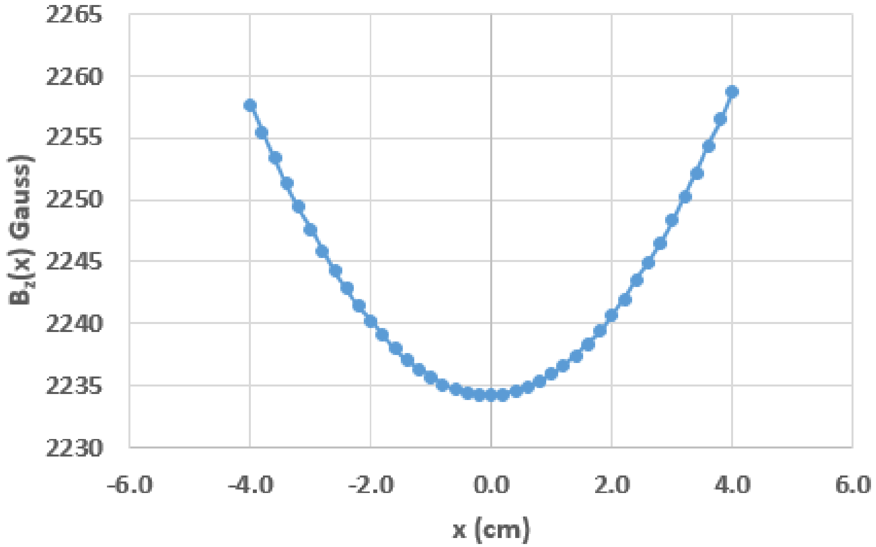
- Spherical aberrations are inherent in any finite length solenoid
- In general, the longitudinal field is larger for r>0, or:

$$\int B_z(z, r \neq 0)^2\, dz > \int B_z(z, 0)^2\, dz$$

- Note that the focusing strength of a solenoid is determined by the integral of the square of the longitudinal field



Axis 2 final focus



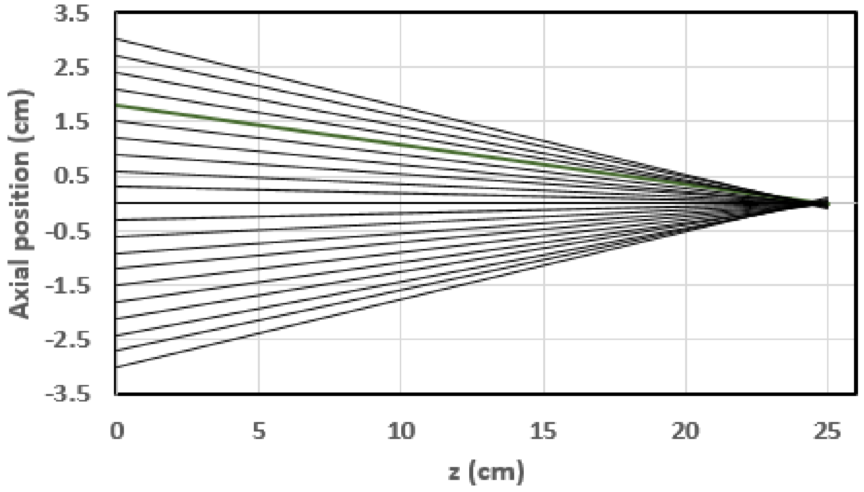
S4

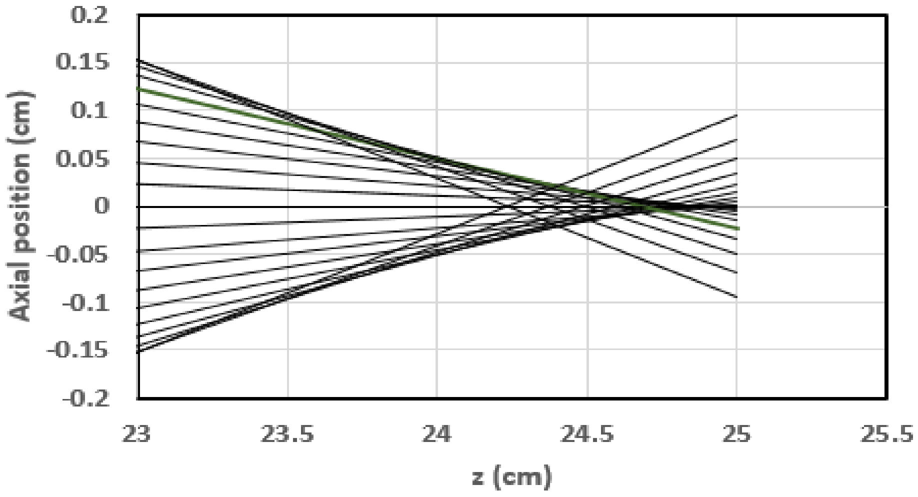
Particles at larger radii encounter a stronger focusing force

- The focal length of a solenoid magnet is given by:

$$f = \left(\frac{2\gamma\beta m_e c}{e}\right)^2 \frac{1}{\int B_z{}^2(z,0)\,dz} = \frac{1}{K}\frac{1}{\int B_z{}^2(z,0)\,dz}$$

- We can calculate the trajectories of a cold (ε=0) beam in the paraxial approximation
- For a final focus solenoid centered 25 cm from the target (like Axis 2)





- This demonstrates that spherical aberrations can significantly alter both the spot size on target and the optimal field in the final focus magnet required to produce the smallest spot

Many factors contribute to the spot size on target

- Beam emittance
- Chromatic aberrations due to the finite energy spread of the beam
- Spherical aberrations due to the final focus solenoid
- Beam motion
  - BBU, kicker smear, etc. for individual pulses
  - Beam motion between pulses for multi-pulse targets
- Beam target dynamics
  - Backstreaming ions in single pulse targets
  - Evolution of vaporized target material in multi-pulse targets
- Finite target thickness
- Non-linear beam distributions

We will focus on the first three (emittance, chromatic and spherical aberrations)

Contributions to the target spot size (emittance and energy spread)

- Emittance

$$R_f \cong \frac{\varepsilon}{R_o} f$$

- Chromatic aberrations

$$\frac{\partial R_f}{\partial f} \cong \frac{\varepsilon}{R_o} \quad , \quad \frac{\partial f}{\partial \gamma} \cong \frac{2f}{\gamma} \quad \rightarrow \quad \Delta R_f = 2R_o \frac{\Delta\gamma}{\gamma}$$

- Adding in quadrature

$$R_f^2 = \left(\frac{\varepsilon}{R_o} f\right)^2 + \left(2R_o \frac{\Delta\gamma}{\gamma}\right)^2$$

Contributions to the target spot size (spherical aberrations)

- There are many different treatments of the effect of spherical aberrations on the focused spot size in the literature

$$R_f^2 = \left(\frac{\varepsilon}{R_o} f\right)^2 + \left(2R_o \frac{\Delta\gamma}{\gamma}\right)^2 + (C_S R_0^3)^2$$

- Chen $$C_S = \frac{1}{\sqrt{2}} \frac{\int B_{z0} B_{z0}'' dz}{\int B_{z0}^2 dz}$$

- El-Kereh $$C_S = \frac{1}{2} \frac{\int {B_{z0}'}^2 dz}{\int {B_{z0}}^2 dz}$$

- Allen calculated the emittance growth due to spherical aberrations

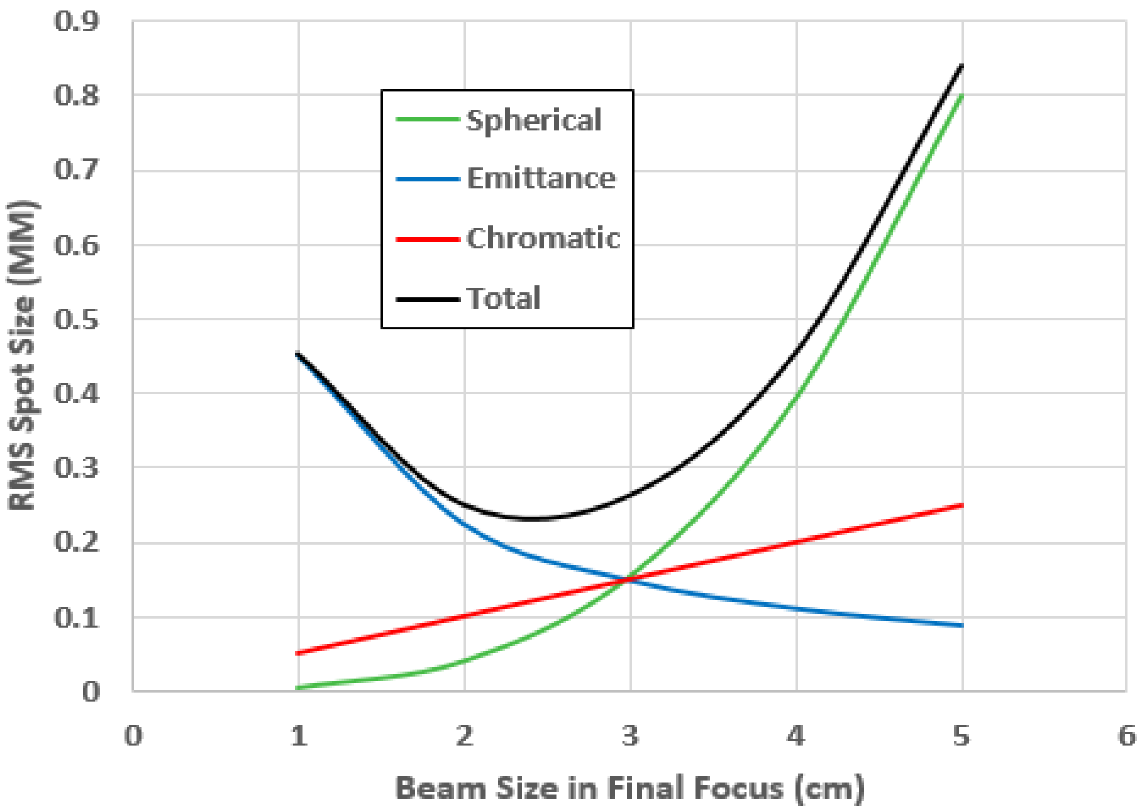
$$\varepsilon_r = \frac{R^4}{2\sqrt{6} f} \sqrt{\frac{C_1^2}{12} + \frac{C_1 C_2}{5} R^2 + \frac{C_2^2}{8} R^4} \text{ where } C_1 = \frac{1}{2} \frac{\int {B_{z0}'}^2 dz}{\int {B_{z0}}^2 dz},\ C_2 = \frac{5}{64} \frac{\int {B_{z0}''}^2 dz}{\int {B_{z0}}^2 dz}$$

- Regardless, spherical aberrations result from the radial variation of the longitudinal solenoid field
  - To first order $\int {B_z}^2(z,r)\, dz = (1 + \alpha r^2) \int {B_z}^2(z,0)\, dz$
  - Minimize α to minimize spherical aberrations

See LA-UR-20-22042 for details and references

Relative contributions to spot size

- Spot size is dominated by emittance at smaller R_0
  - Unfortunately, this also leads to larger spots
- For large R_0, spot size is dominated by spherical aberrations
- Always minimize energy spread
- The optimal design of the final focus involved minimizing the focal length and C_S.



Optimal beam size for a specific final focus system

- Taking the derivative of

$$R_f^2 = \left(\frac{\varepsilon}{R_o} f\right)^2 + \left(2R_o \frac{\Delta\gamma}{\gamma}\right)^2 + (C_S R_0^3)^2$$

with respect to R_0 will give the optimal beam size in the final focus magnet

$$3C_S^2 R_0^8 + 2\left(\frac{\Delta\gamma}{\gamma}\right)^2 R_0^4 - (\varepsilon f)^2 = 0$$

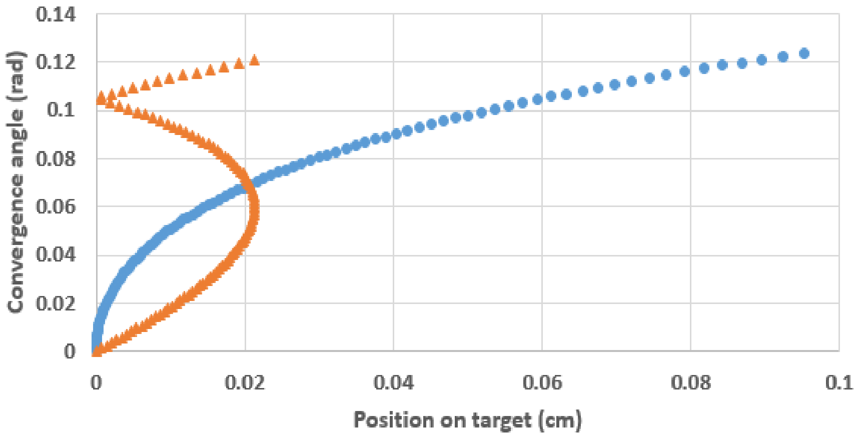
$$\rightarrow R_0^4 = \frac{\sqrt{\left(\frac{\Delta\gamma}{\gamma}\right)^4 + 3(C_S \varepsilon f)^2} - \left(\frac{\Delta\gamma}{\gamma}\right)^2}{3C_S^2}$$

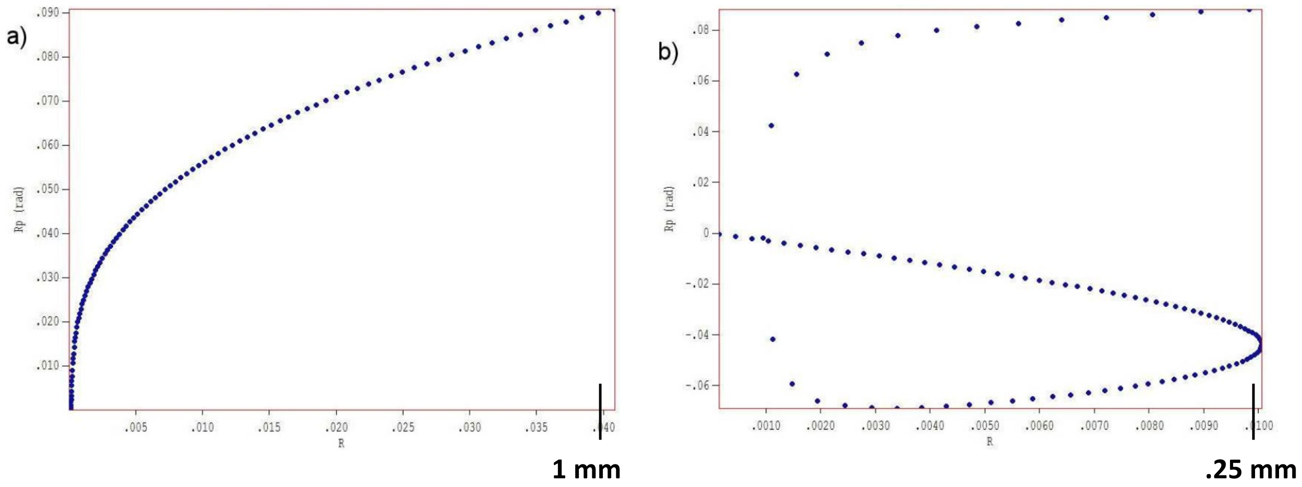
- For very small energy spreads (Axis 2 P1-P3)

$$R_0^4 \cong \frac{\varepsilon f}{\sqrt{3} C_S} - \frac{1}{3C_S^2}\left(\frac{\Delta\gamma}{\gamma}\right)^2$$

CAUTION!!!

- Using the value of C_S given previously will overestimate the contributions of spherical aberrations to the minimum by over a factor of four!
  - **The strength of the final focus magnet must be reduced from the value calculated on the basis of emittance only to achieve the smallest spot size**

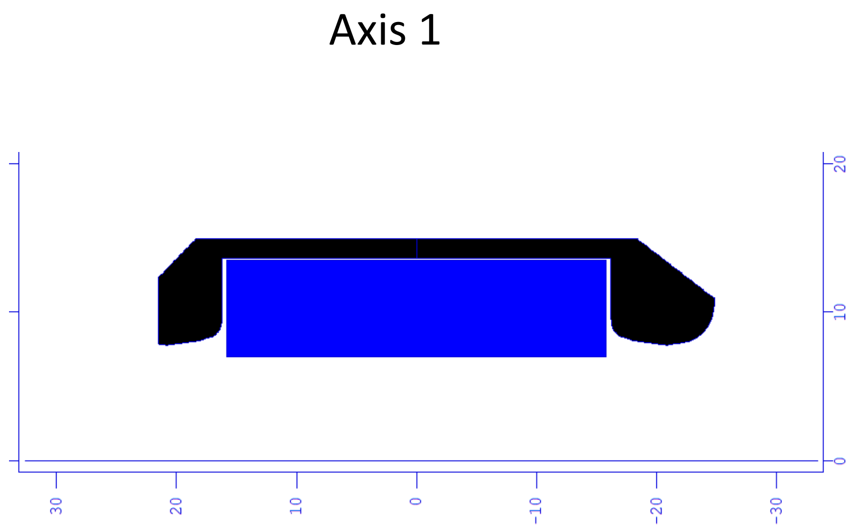


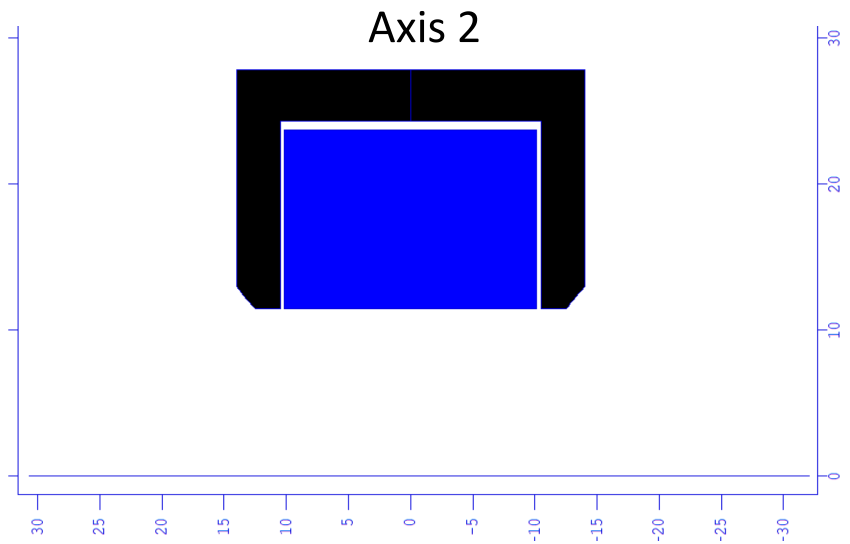


Phase space plot of cold beam on target with blue dots corresponding to paraxial rays and orange triangles for reduced final focus field

Phase space plot of a nominal beam (ε_n=1000π(mm-mrad)) on target by Humphries in review of the Axis 2 final focus magnet. The current in the final focus magnet was reduced from 438.3 A to 432.5 A for (a) and (b) respectively

Present Axis 1 and Axis 2 Final Focus Solenoids

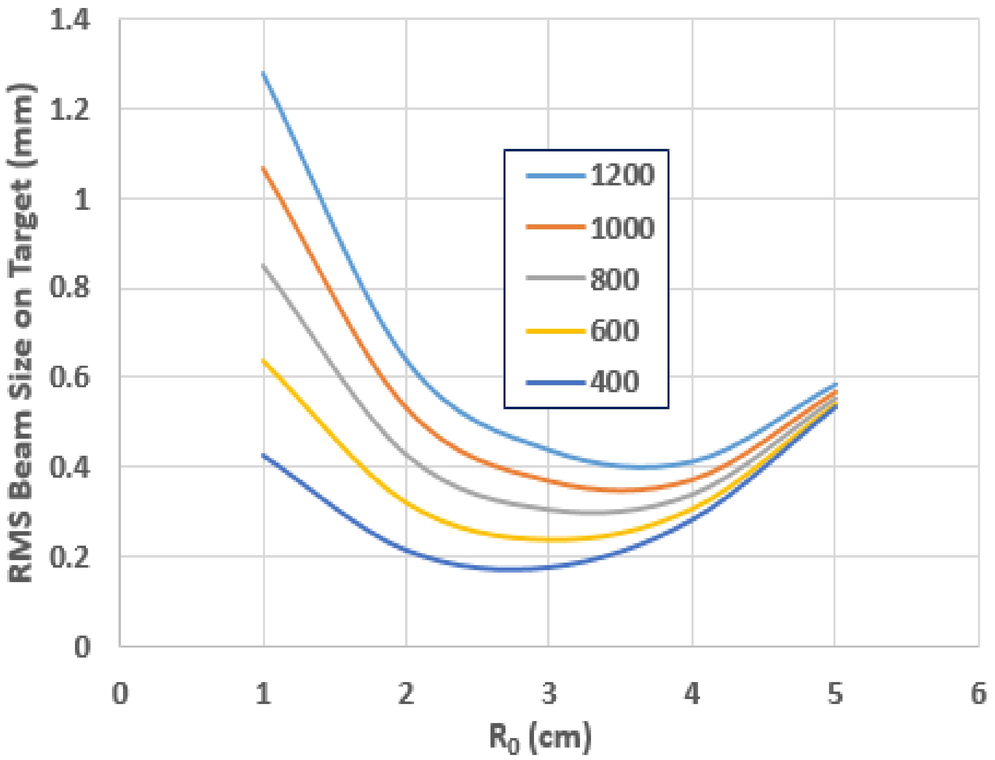




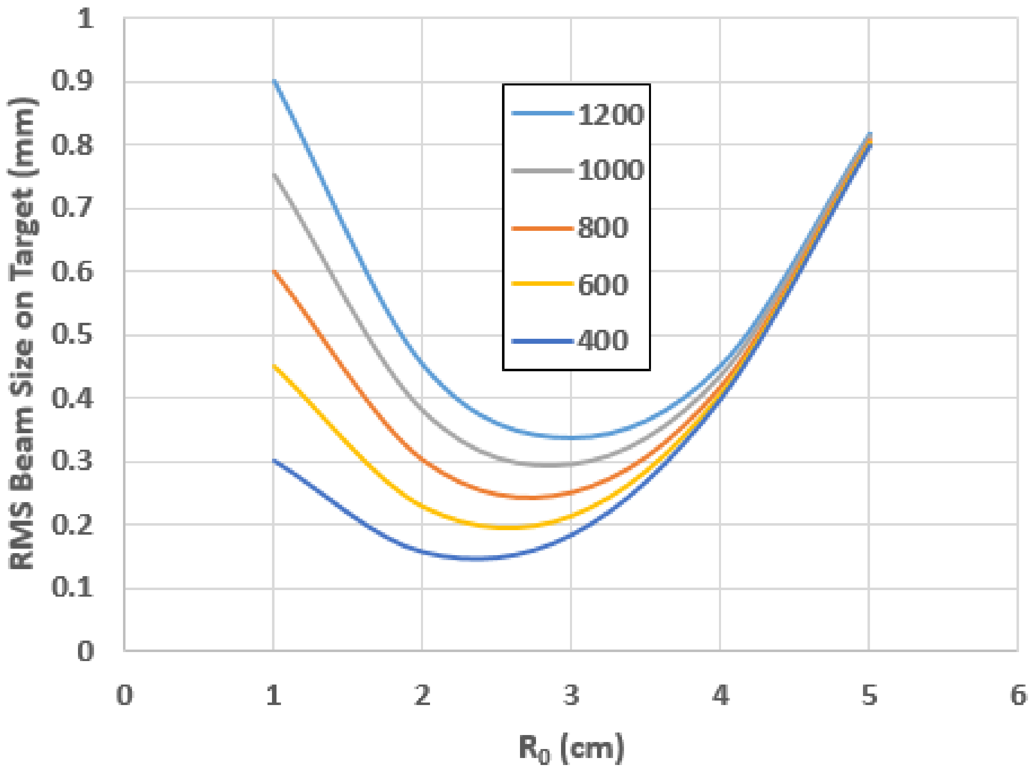
- Magnet cross sections are shown on same scale with black corresponding to the steel yoke and blue representing the coil
  - Axis 1 is longer with a smaller radius
    - The truncated pole is on the target side and reduces the focal length
    - Note that the coil ID is smaller than the steel ID
  - Axis 1 (C_S= -.00275 cm^{-2}, α=.00105 cm^{-2})
  - Axis 2 (C_S= -.00352 cm^{-2}, α=.00184 cm^{-2})

Comparison of target spot sizes ($\Delta E=0$)

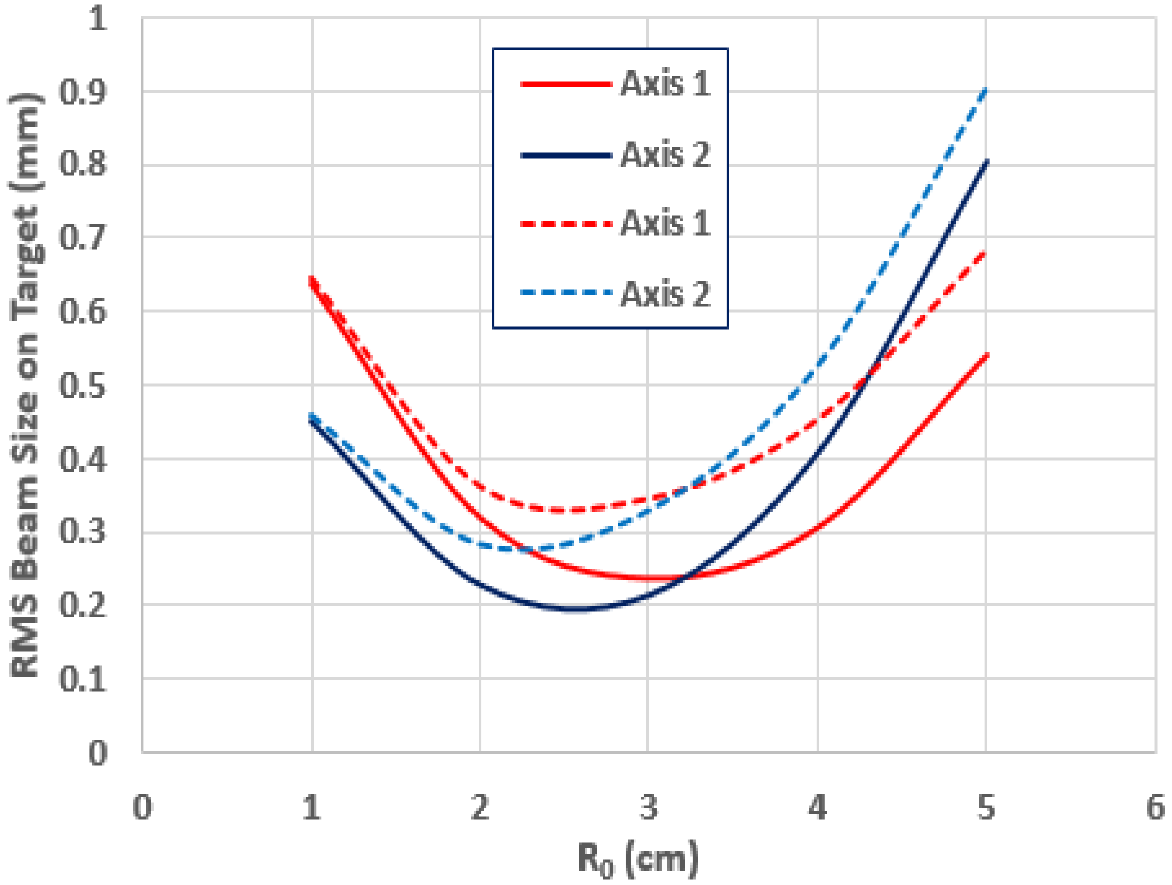
Axis 1



Axis 2



Comparison of target spot sizes

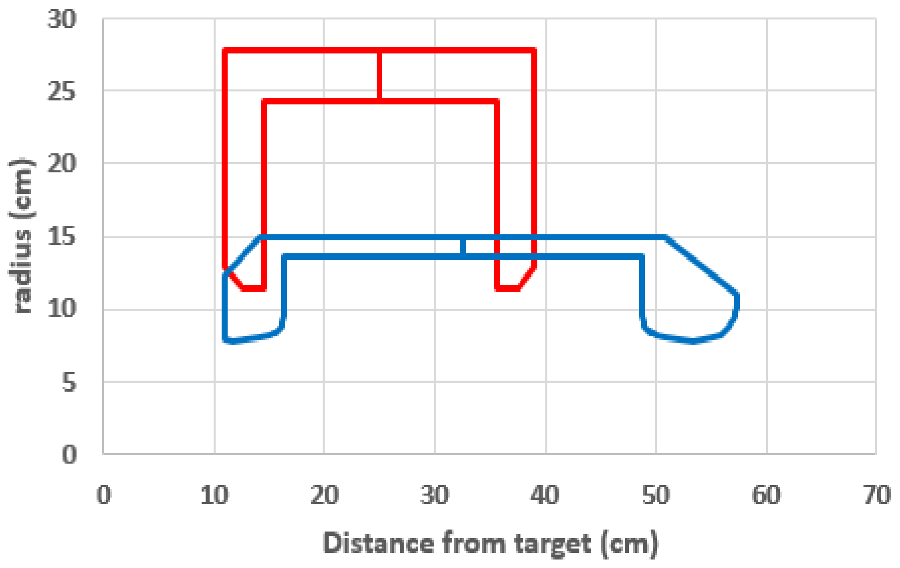


ε_n = 600 π(mm-mrad) with (dashed) and without (solid) an energy spread of 1% FWHM

- The advantages of the shorter focal length of Axis 2 and the reduced spherical aberration of Axis 1 are clearly evident in this figure.
- Can we design a new solenoid with both features?
- Down the rabbit hole

Detailed comparison of Axis 1 and Axis 2 final focus solenoids

- In general, spherical aberrations are reduced by increasing length and radius.
- Minimizing length reduces the focal length
- Plan (Start with Axis 2 design)
  - Investigate Axis 1 “like” upstream pole
  - Increase radius
  - Investigate pole ID greater than coil ID
  - Iterate, iterate, etc
  - Increase OD to accommodate required ampere-turns



Cross sectional views of Axis 1 (blue) and Axis 2 (red) steel yokes showing magnet orientation with respect to the target. The vertical line in the center of each yoke is the approximate focal length.

After many iterations

Model	Description
Nominal	Existing Axis 2 Magnet
Mod7a	Axis 2 with Axis 1 upstream pole at coil radius
Mod7	Axis 2 with Axis 1 upstream pole at larger radius than coil like Axis 1
Mod8	Same as Mod7 with modified downstream pole and 2 cm radius increase
Mod6	Same as Mod8 with 2 cm length increase and simplified upstream pole
Mod9	Same as mod6 with 2 cm radius increase and thicker downstream pole
Mod5a	Same as Mod6 with 2 cm length decrease
Mod5b	Same as Mod5a with upstream pole optimization

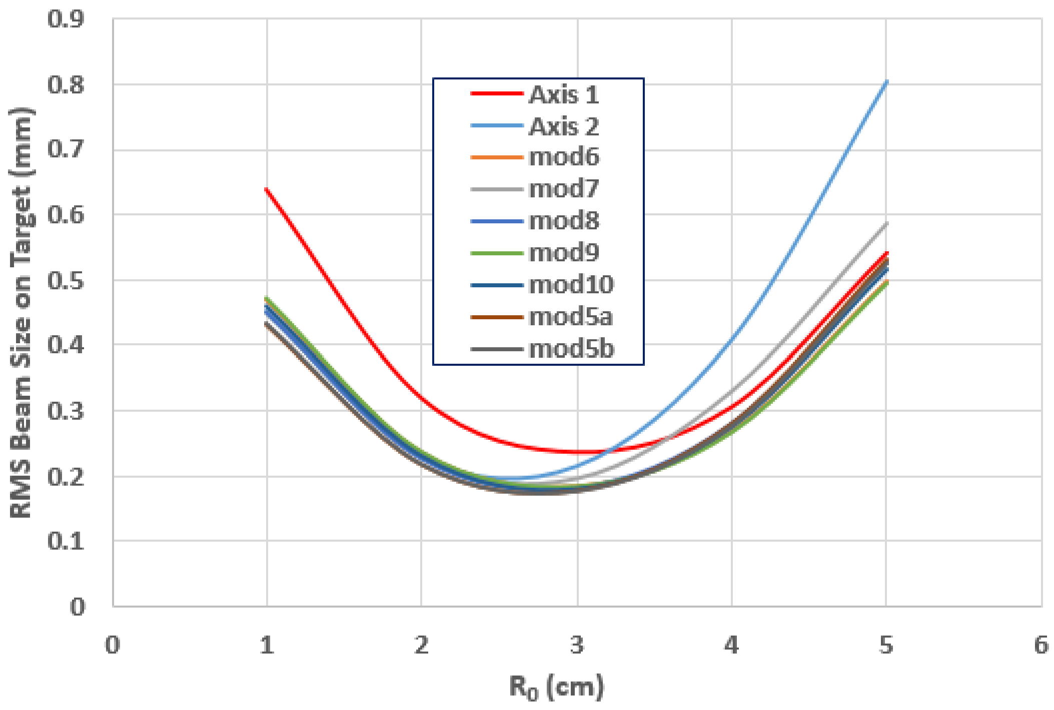
Model	Focal Length (cm)	C_s (1/cm^2)	α (1/cm^2)	L_{eff} (cm)^1	L_{eff} (cm)^2	B_0 (Gauss)	B/NI (Gauss/A)	Current (A)
Axis 1	42.53	-0.00275	0.00105	33.21	26.96	4043	0.0379	445.0
Axis 2	25	-0.00352	0.00184	26.95	19.24	5177	0.0473	456.2
mod7a	25	-0.00341	0.00130	26.58	19.04	5204	0.0471	460.3
mod7	25	-0.00329	0.00125	27.02	19.25	5176	0.0463	465.5
mod8	25	-0.00273	0.00095	29.44	20.48	5017	0.0424	493.2
Mod6	26	-0.00255	0.00095	30.58	21.44	4809	0.0408	490.7
mod9	26.25	-0.00255	0.00095	30.64	21.48	4759	0.0407	486.9
mod5a	24	-0.00276	0.00103	29.26	19.87	5200	0.0426	508.6
mod5b	24	-0.00271	0.00101	29.48	20.09	5171	0.0423	509.5

1 - $\int B dz / B_0$

2 - $\int B^2 dz / B_0^2$

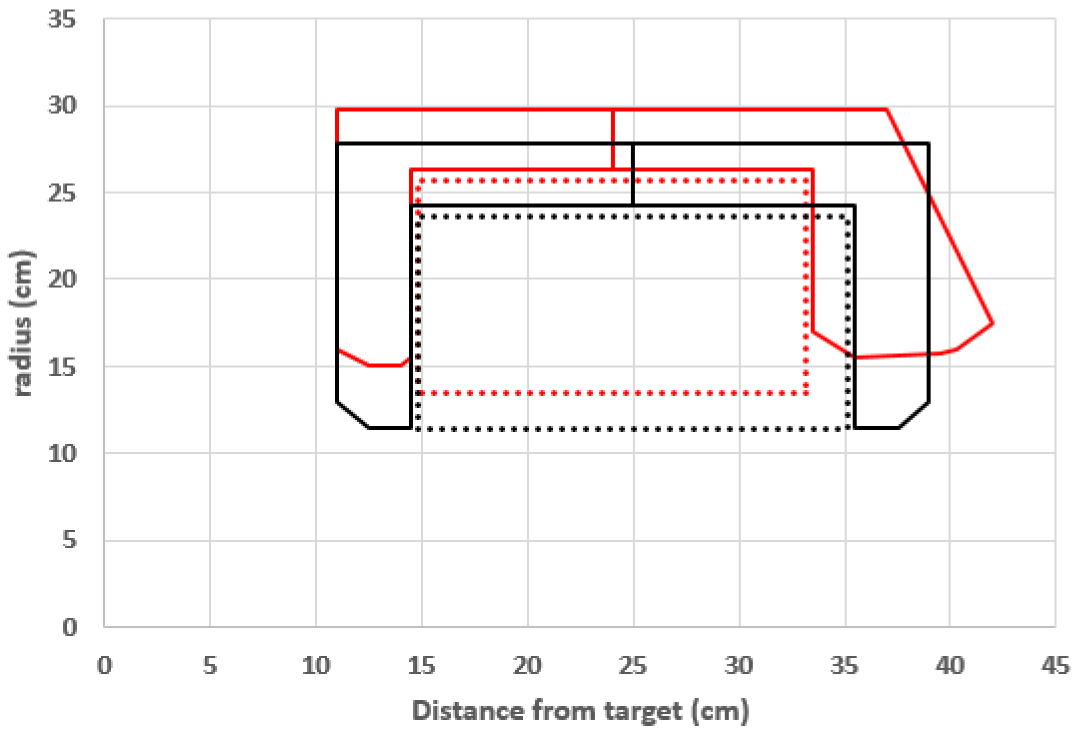
Improvement in the minimum spot size

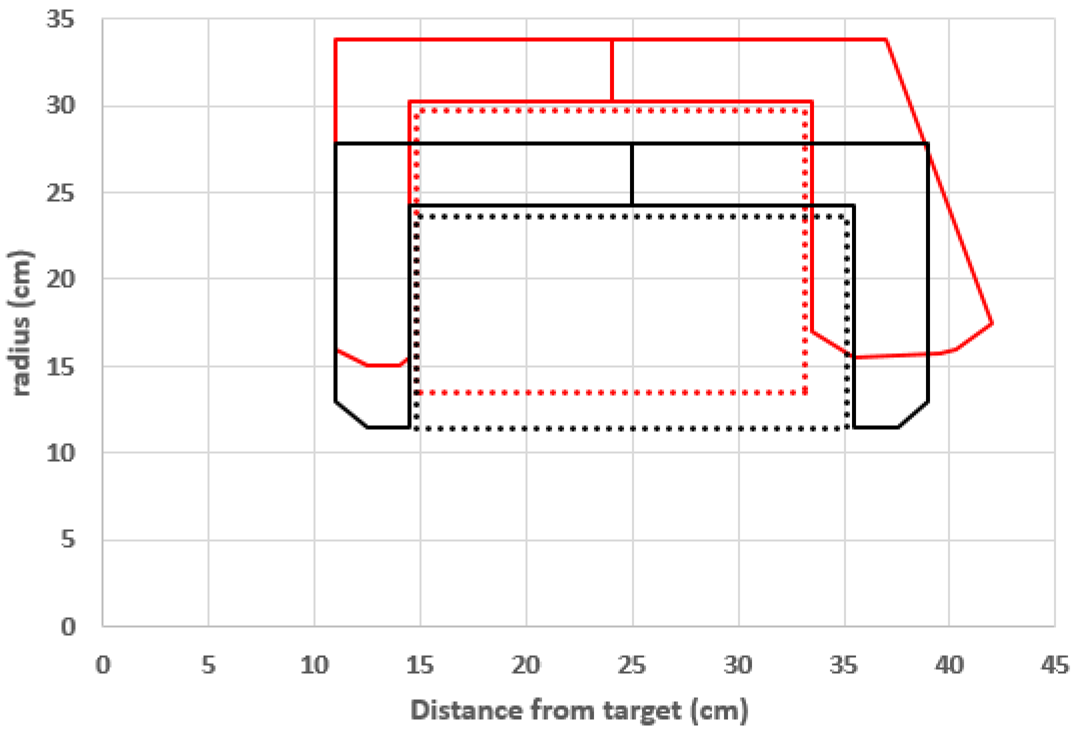
- As spherical aberrations were reduced the slope at larger R_0 was reduced
- The overall minimum spot size was reduced by 26% and 17% respectively for Axis 1 and Axis 2 for a normalized beam emittance of 600 π(mm-mrad)



Modify mod5b for more coil cross section

- The spherical aberration coefficient changes by less than 1%
- Note the coil cross section is smaller for the design on the left



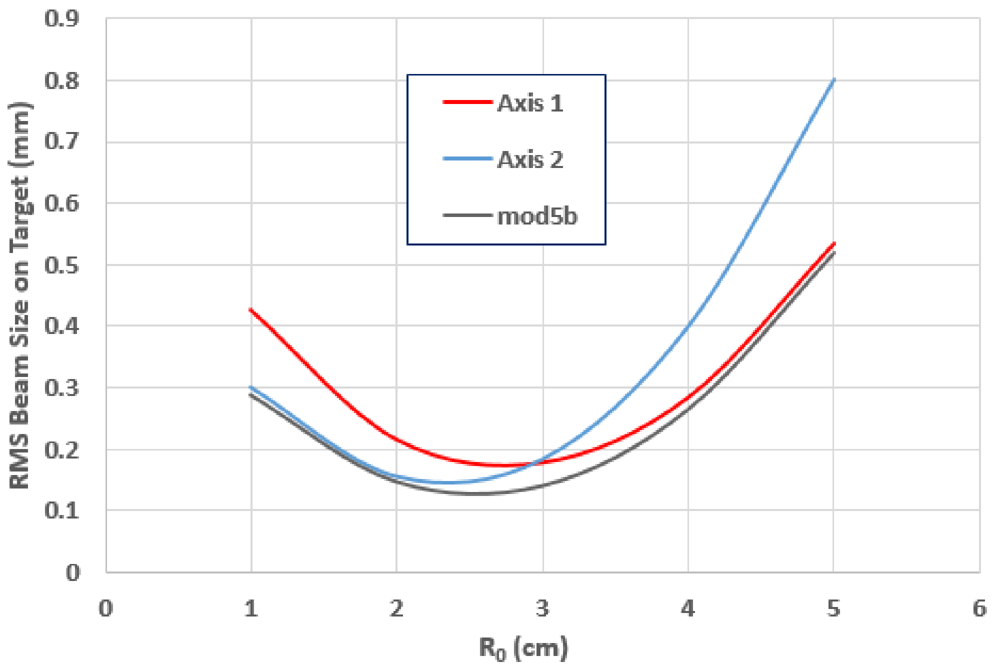


Electro-mechanical parameters

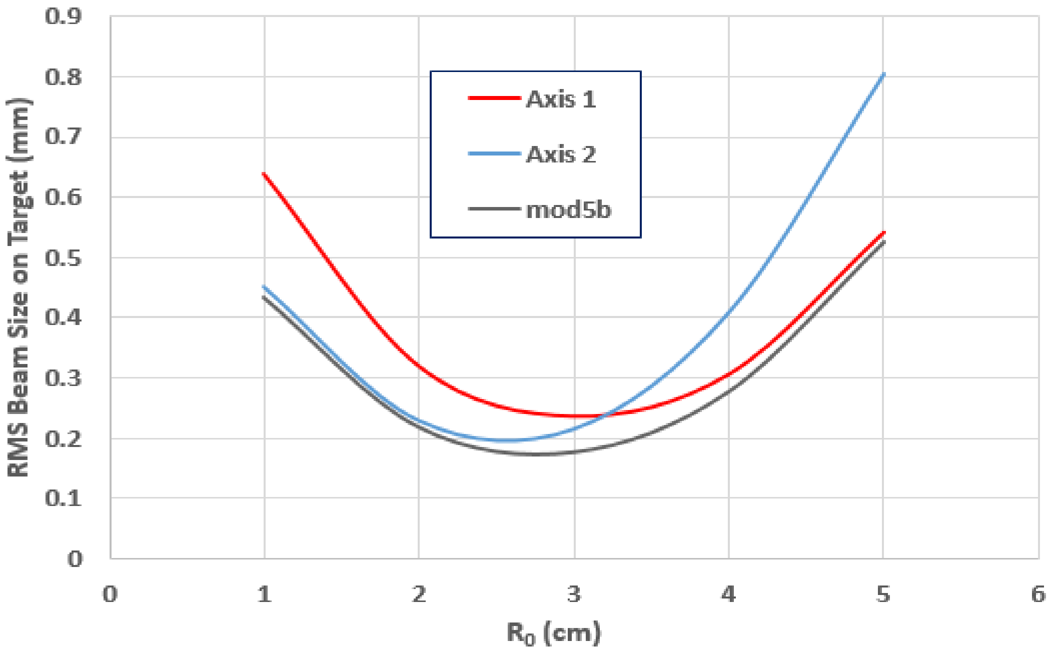
Parameter	units	Axis 2	mod5b		larger conductor ID		more turns	
Beam Energy	(MeV)	16.5	16.5	20.0	16.5	20.0	16.5	20.0
Magnetic Field	(kG)	5.449	6.103	7.360	6.103	7.360	6.103	7.360
Current	(A)	480	597	720	597	720	448	540
turns		20	18	18	18	18	18	18
layers		12	12	12	12	12	16	16
Total Turns	N	240	216	216	216	216	288	288
Conductor size	(square)	0.325	0.325	0.325	0.325	0.325	0.325	0.325
Conductor Hole	(in)	0.181	0.181	0.181	0.225	0.225	0.225	0.225
Resistance	(mOhm)	95.9	96.1	96.1	122.8	122.7	171.7	171.7
Voltage	(V)	46.0	57.4	69.3	73.4	88.4	76.9	92.8
Power	(kW)	22.1	34.3	49.9	43.8	63.7	34.5	50.1
Pressure Drop	(psi)	60	60	60	60	60	60	60
Total Flow	(gpm)	14.3	12.8	12.8	22.2	22.2	19.0	19.0
Tempurature Rise	(deg C)	5.9	10.2	14.8	7.5	10.9	6.9	10.1

The last two columns required a 4 cm increase in the yoke OD

Spot size improvement depends on emittance



The overall minimum spot size was reduced by 39% and 14% respectively for Axis 1 and Axis 2 for a normalized beam emittance of 400 π(mm-mrad)



The overall minimum spot size was reduced by 26% and 17% respectively for Axis 1 and Axis 2 for a normalized beam emittance of 600 π(mm-mrad)

This reflects the relative importance of improving the spherical aberrations vs reducing the focal length

Spherical aberrations and solenoid scans

- Now that we have examined the effect of spherical aberrations on the target spot size, let's try to understand the impact of spherical aberrations on a solenoid scan.
  - Solenoid scans are often used to infer the emittance of an intense relativistic electron beam (IREB).
  - Beam size is measured at a location downstream of the solenoid as a function of the strength of the solenoid magnetic field.
  - The emittance is then inferred using a beam optics model of the transport of the beam through the solenoid magnet to the imaging location.
    - Precise knowledge of the field strength and shape as a function of the current supplied to the solenoid is required.
    - The beam optics model for an IREB needs to include an accurate model of the magnetic field.
    - Most beam optics models **do not include the effect of spherical aberrations** which are inherent in a solenoid magnet.
- We examine the sensitivity of a typical solenoid scan performed on DARHT Axis 2 to spherical aberrations.
  - The sensitivity to spherical aberrations is examined as a function of the beam envelope size in the solenoid and the drift distance from the solenoid magnet to the imaging station.
  - The initial beam emittance is also varied.

Paraxial thin lens model

- Focal length

$$f = \left(\frac{2\gamma\beta m_e c}{e}\right)^2 \frac{1}{\int {B_z}^2(z,0)\,dz} = \frac{1}{K}\frac{1}{\int {B_z}^2(z,0)\,dz}$$

- Radial dependence of B

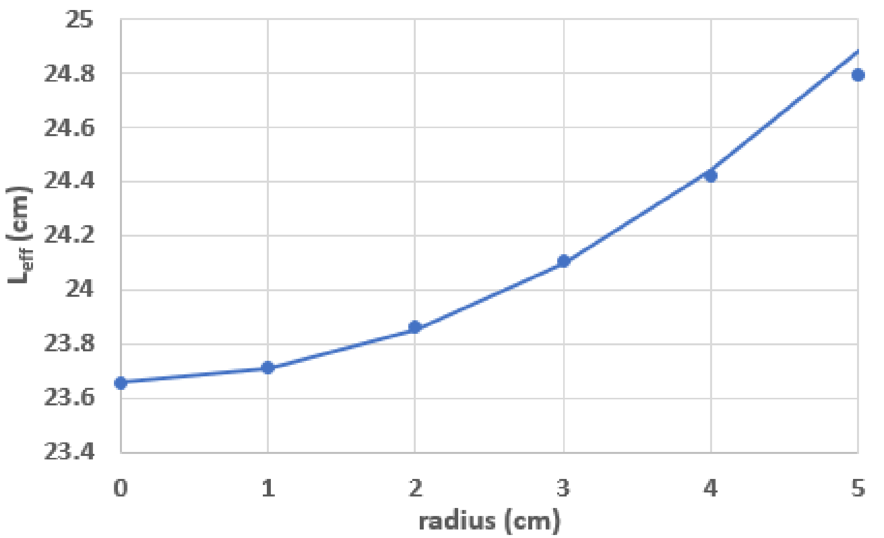
$$B_z(z,r) = B_z(z,0) - \frac{r^2}{4}\frac{\partial^2 B_z(z,0)}{\partial z^2} + \cdots$$

- Radial dependence of focal length

$$f(r) = \frac{1}{K}\frac{1}{B_0^2 L_{eff}(r)}$$

- Radial dependence of solenoid effective length

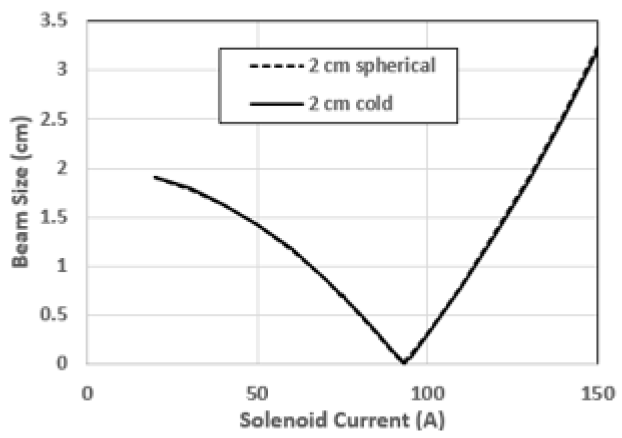
$$L_{eff}(r) = L_{eff}(0)(1 + \alpha r^2)$$

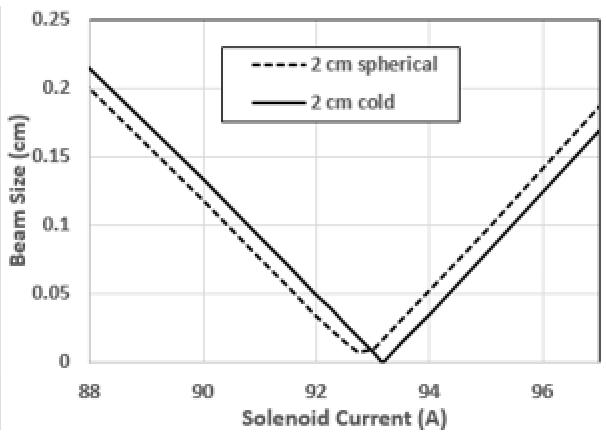


- For the Axis 2 DST solenoids, α=.002071 cm^{-2} which is almost twice the proposed final focus magnet

Compare beam envelope for cold beams with and without spherical aberrations

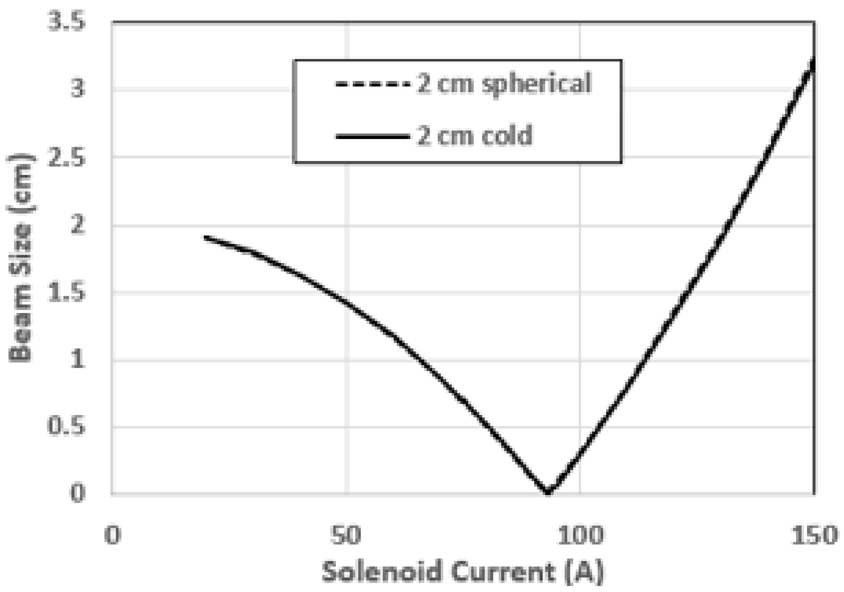
- Trace hundreds of rays from an initial beam location.
- The beam size as a function of the drift distance is then determined from the ray with the maximum radial extent.
- Repeat for many different values of the solenoid field or current.
- The results are compared to cold beam without spherical aberration
- Example:
  - 2 cm initial beam radius
  - Beam size is plotted as a function of solenoid current at a distance 1.0 m from the magnet

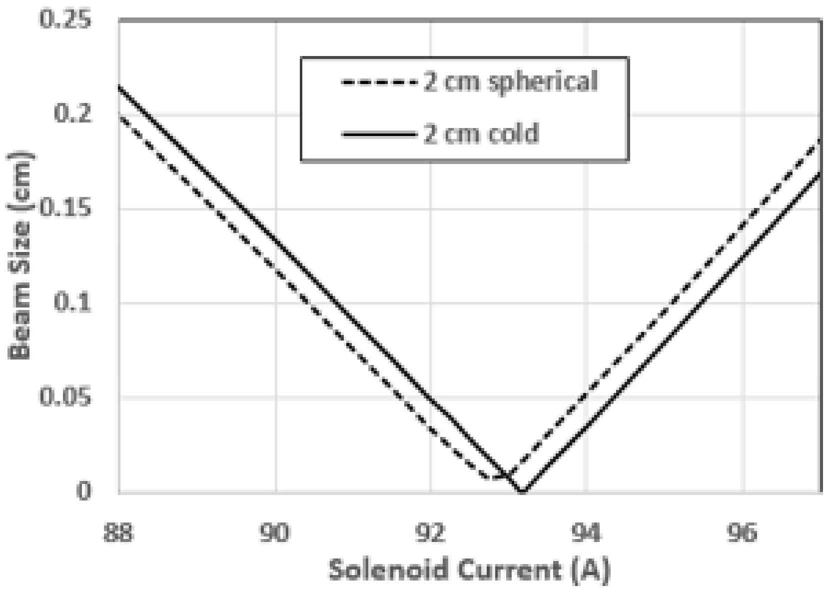




Spherical Aberrations and Solenoid Scans (1)

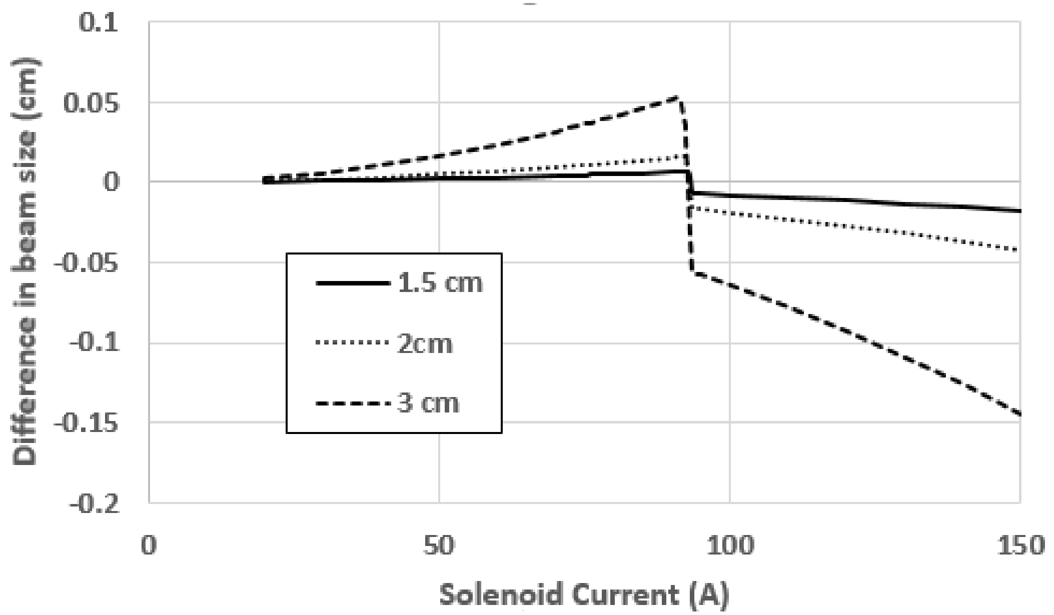
- Effect of spherical aberrations for a beam size of 2.0 cm and a drift distance of 1.0 m from the solenoid to the imaging station
  - Beam is smaller at low current prior to focus
  - Focus occurs at lower currents
  - Beam is larger at focus
  - Beam is larger at higher currents





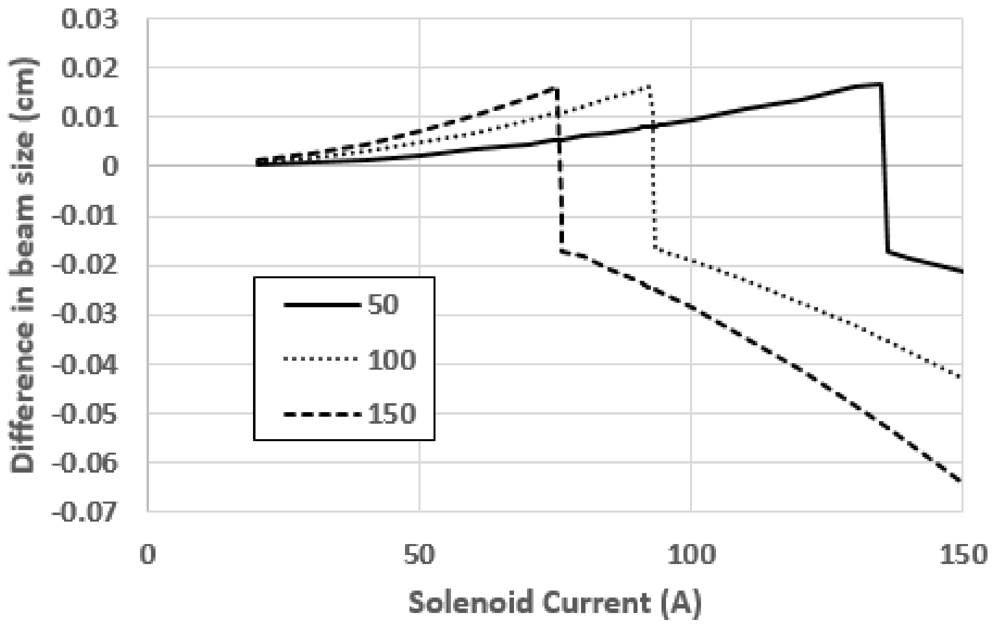
Spherical Aberrations and Solenoid Scans (2)

- Contribution of spherical aberrations as a function of spot size for a drift distance of 1.0 m from the solenoid to the imaging station
  - The contribution of spherical aberrations goes as the square of the beam size in the solenoid as expected
  - The size of the correction at the beam focus is about 0.5 mm for an initial beam size of 3.0 cm



Spherical Aberrations and Solenoid Scans (3)

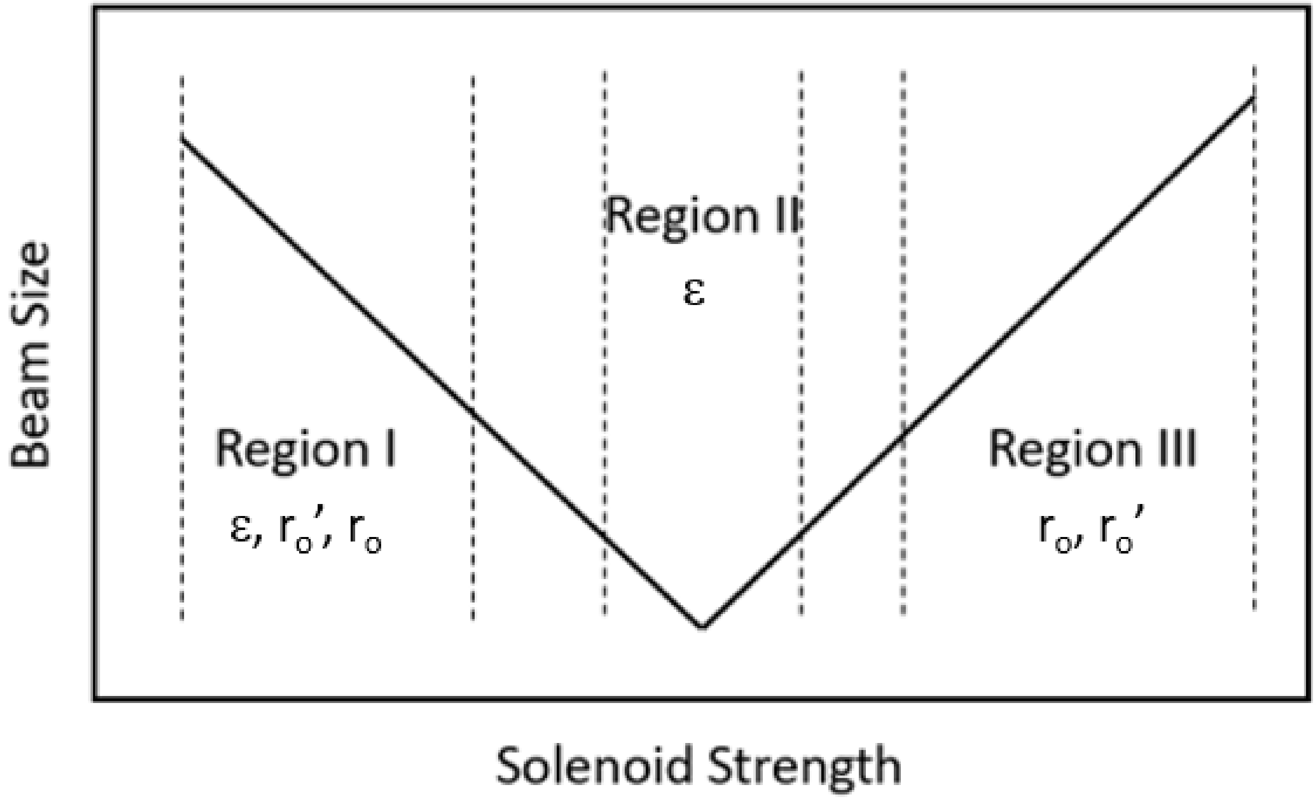
- Effect of spherical aberrations for a beam size of 2.0 cm as a function of the drift distance from the solenoid to the imaging station
  - The contribution of spherical aberrations at the focus is essentially independent of the distance from the solenoid magnet to the target.
  - This implies a larger effect on solenoid scans with shorter drifts from the solenoid because the beam size at the focus is smaller for smaller focal lengths.
  - Overall, the contribution of spherical aberrations is much less for longer drift distances
  - In other words: **Carl was right!**



How to include these effects in a simulated solenoid scan and estimate the change in emittance

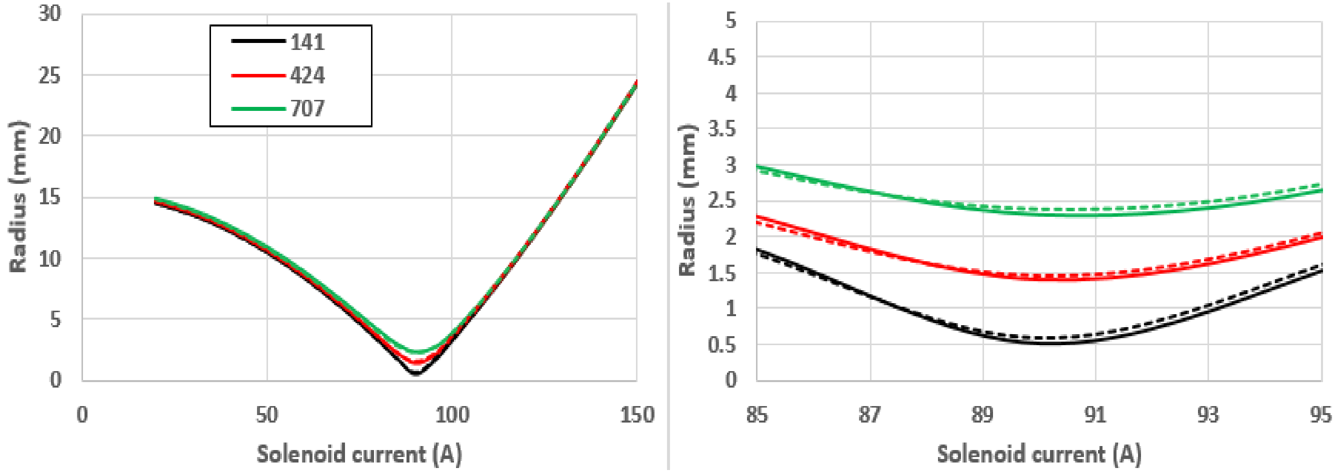
- Accurate calculations of the effect of spherical aberrations on a solenoid scan would require hundreds of simulations using a 2D-PIC code which is not done here.
- The prior analysis shows the relative sensitivity of the measured spot size to spherical aberrations in a solenoid scan. The initial beam sizes and drift distances are kept the same as above and the beam size is calculated as a function of the solenoid current for three different values of the normalized rms emittance (141, 424 and 707 π(mm-mrad)).
- Adjust the results of simulated scans performed using TRANSPORT to approximate the effect of spherical aberrations.
- The difference in spot size for a cold beam with and without spherical aberrations will be added to the TRANSPORT calculation for a simulated scan. A comparison of the TRANSPORT results with and without this correction allows for an estimate of the effect of spherical aberrations on a solenoid scan.

Relative sensitivities to inferred beam parameters in a solenoid scan

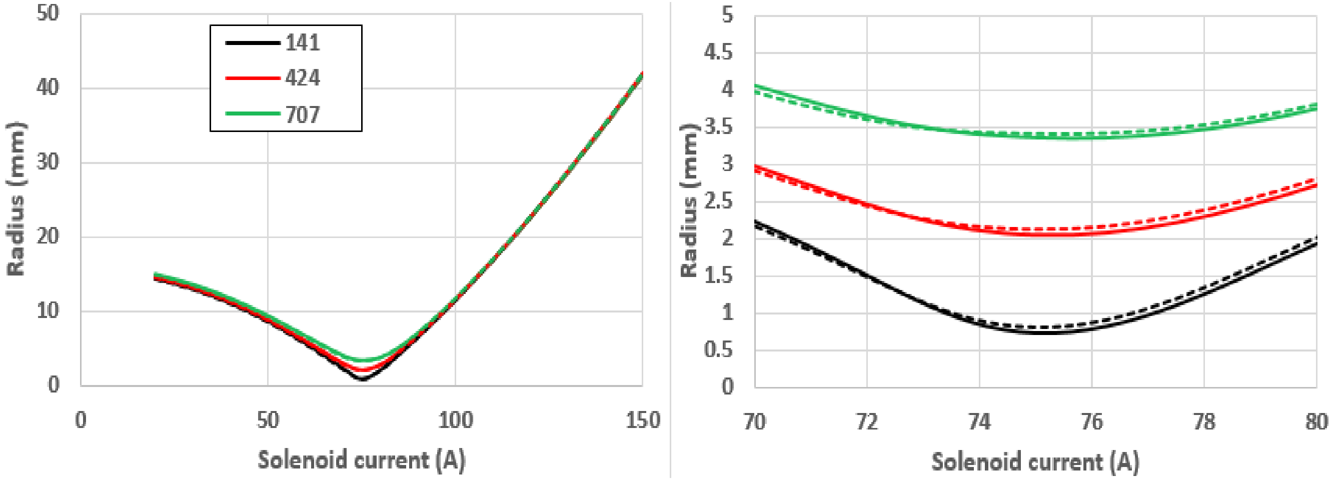


- Region 1 is most sensitive to ε, r_o' and to a lesser extent r_o.
- Region 2 is sensitive to ε and relatively insensitive to r_o and r_o'
- Region 3 is sensitive to r_o and r_o' and insensitive to ε.

Results for an initial beam size of 1.5 cm



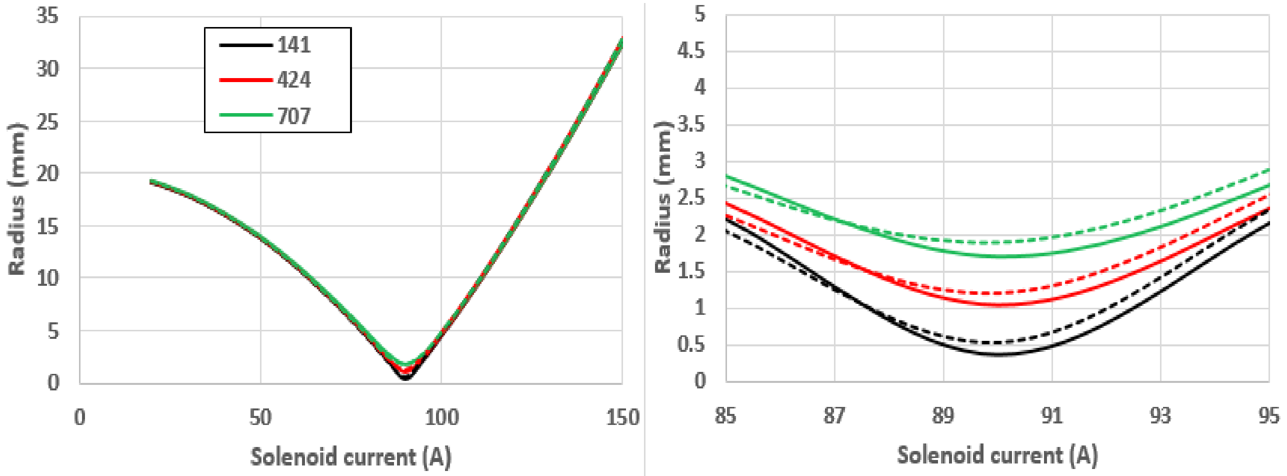
1.0 meter drift



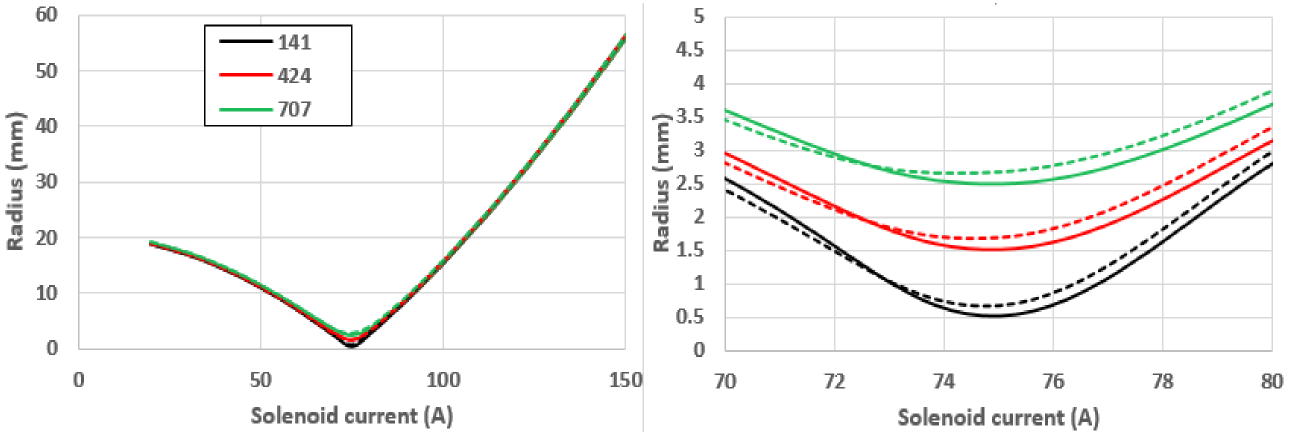
1.5 meter drift

- Insensitivity to emittance in Region 3 is clear
- Increase to beam size at minimum is fairly small
- Increase to beam size at minimum is essentially the same for both drifts
- Relative increase to beam size for longer drift is smaller

Results for an initial beam size of 2.0 cm



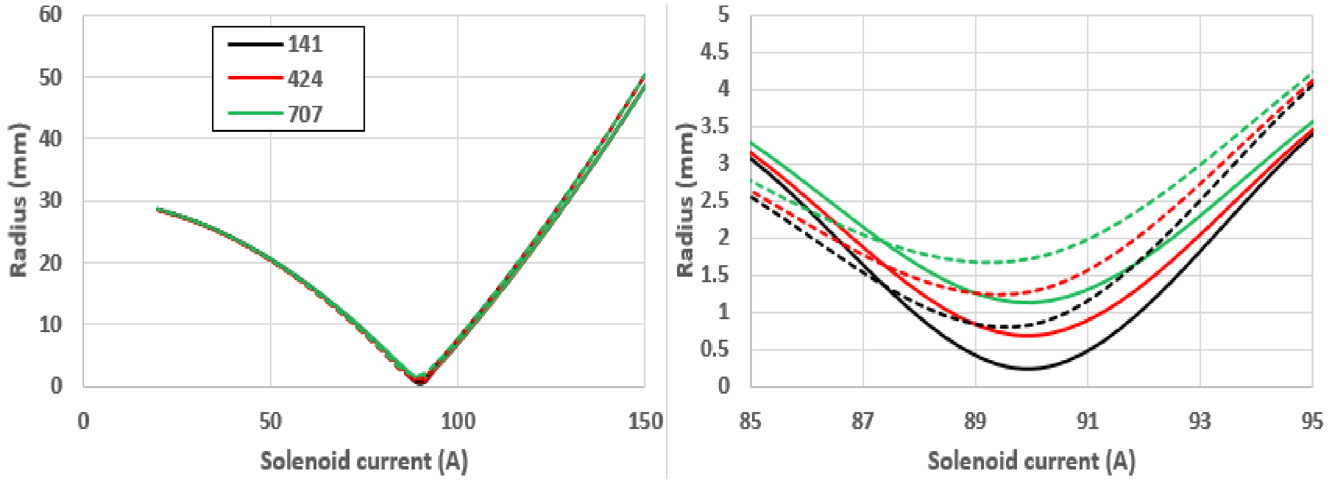
1.0 meter drift



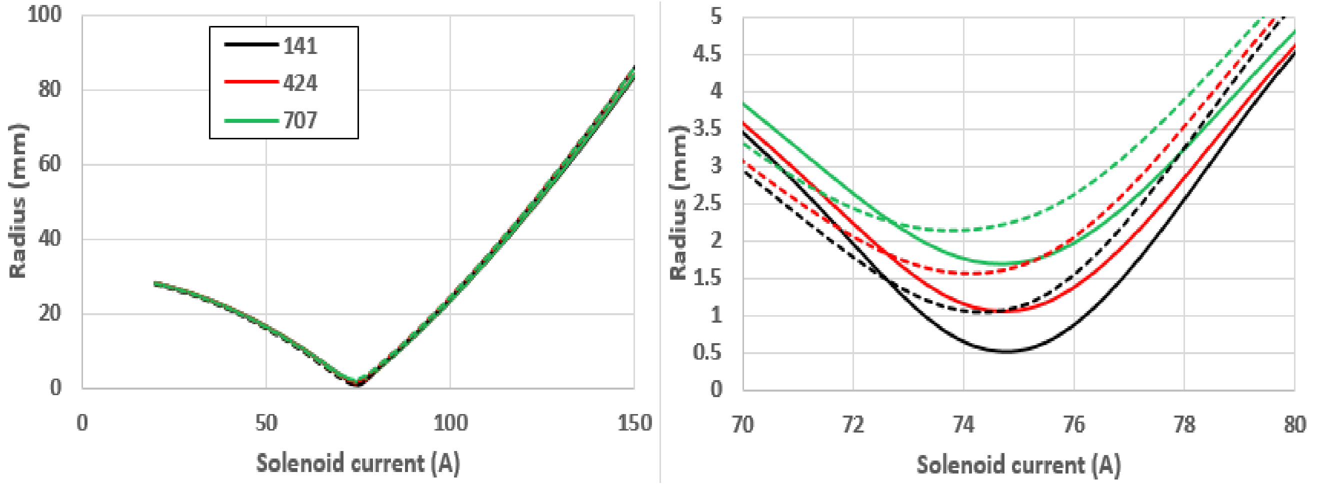
1.5 meter drift

- Insensitivity to emittance in Region 3 is clear
- Increase to beam size at minimum is not small
- Increase to beam size at minimum is essentially the same for both drifts
- Relative increase to beam size for longer drift is smaller

Results for an initial beam size of 3.0 cm



1.0 meter drift



1.5 meter drift

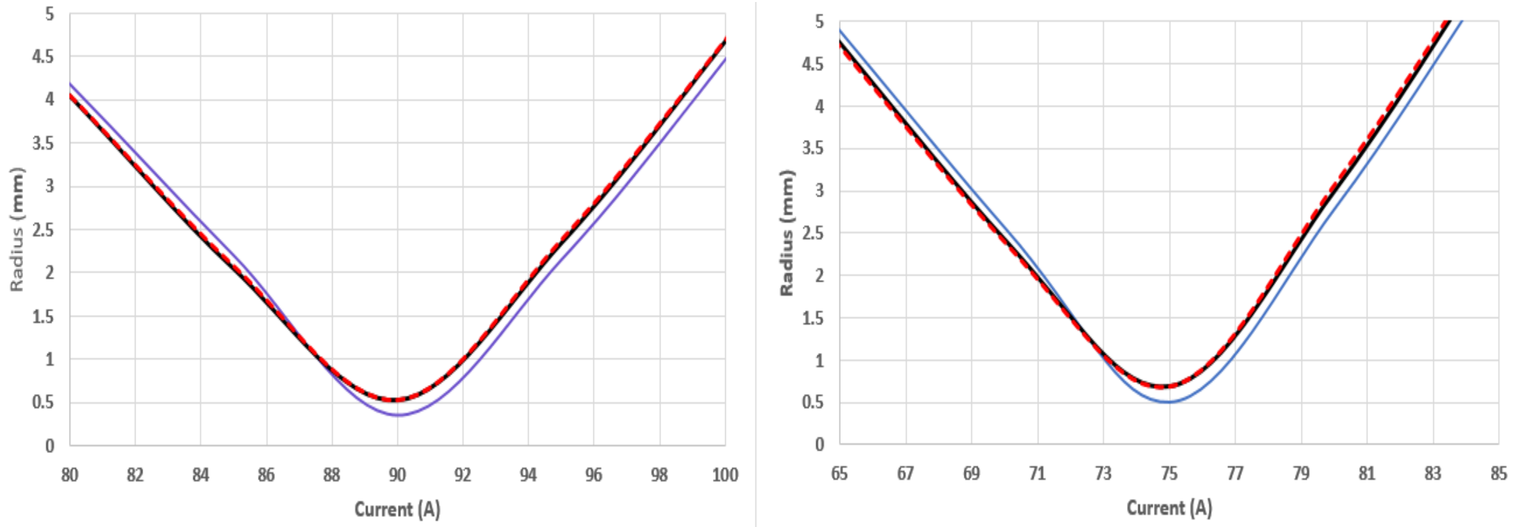
- Insensitivity to emittance in Region 3 is clear
- Increase to beam size at minimum is large
- Increase to beam size at minimum is essentially the same for both drifts
- Relative increase to beam size for longer drift is smaller

Estimated error in inferred emittance

- The error in the inferred emittance can be estimated by taking the ratio of the beam sizes at the minimum
- This estimated rms emittance growth for a 1.0 meter drift is 22, 71, and 360 π(mm-mrad) respectively for initial beam sizes of 1.5, 2.0 and 3.0 cm
- The estimated rms emittance growth for a 1.5 meter drift is about 30% less.

Use TRANSPORT to infer the beam parameters from the modified scan

- A very good fit to the simulated data is obtained
  - The emittance increase is similar to that on the previous slide
    - Approximately 50% increase for a geometry close to that at Station C
  - The increase in the beam convergence is similar for both cases
  - The change in the initial beam size is very small



Parameter	Original beam	Fit at 1.0 m	Fit at 1.5 m
r_o [cm]	2.00	2.03	2.02
r'_o [mrad]	0.00	-0.48	-0.40
ε_{rms} [π(mm-mrad)]	141.42	214.96	178.19

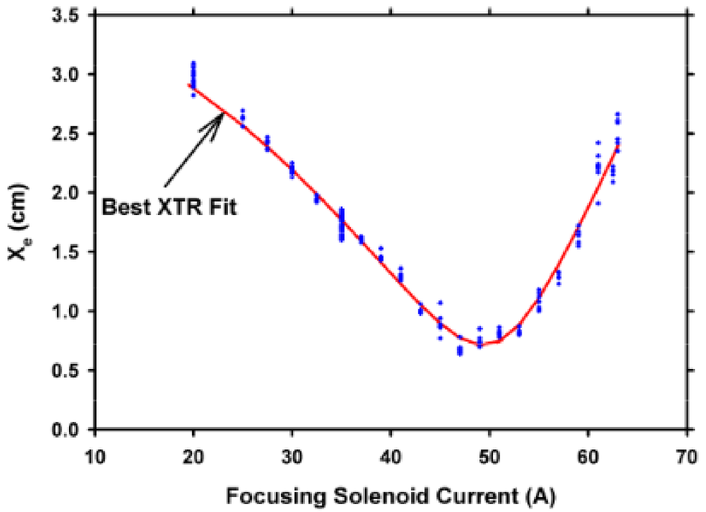
Blue – TRANSPORT **Black – TRANSPORT + difference**

Red dash – TRANSPORT fit to Black "simulated" data

Conclusions

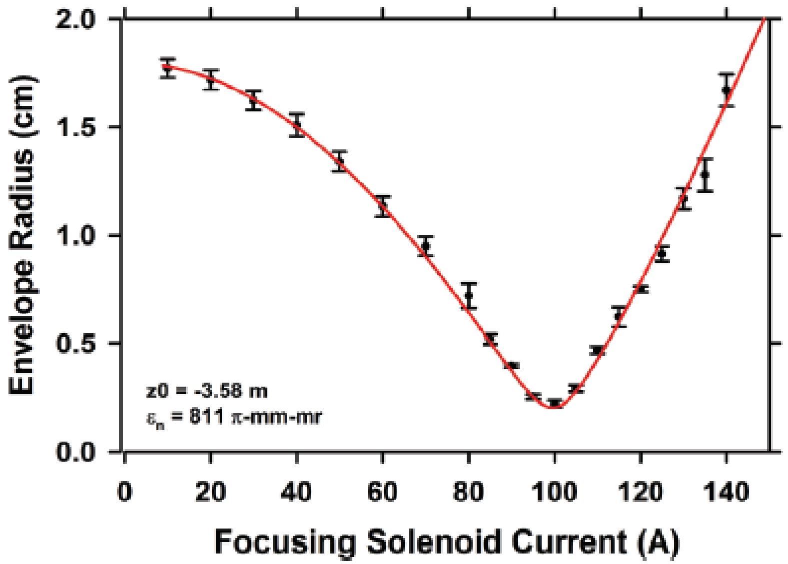
- Spherical aberrations in solenoids are unavoidable and problematic whenever the beam is focused
  - LIA Commandment #? from LLNL
    - Thou shalt not over focus thy beam
- It may be possible to include a spherical aberration correction to the analysis of a solenoid scan in the manner described above although more accurate 2D PIC models should be studied to determine the accuracy of the very simple model used above. This correction would have to be iterated as the beam size in the solenoid is iterated.
- A quadrupole triplet final focus would not have aberrations
  - Preliminary analysis shows improvement in spot size
  - Very difficult to tune

Some solenoid scan results from Axis 2



Parameter		Units	Value	
Energy	KE	MeV	8.02	± 0.5%
Current	I_b	kA	0.9-1.1	± 2%
Radius	R_0	cm	0.8	± 3%
Divergence	R_0'	mr	3.2	± 16%
Normalized Emittance	ε_n	π-mm-mr	617	± 10%

2006 – Scaled accelerator
Station A - 1.76 meters from S1 center
S1 beam radius - ~1.0 cm



Circa 2013 – DST after kicker
Station C - - 1.08 m from S4 center
S4 beam radius - ~ 2.0 cm

Comparison of solenoid scans

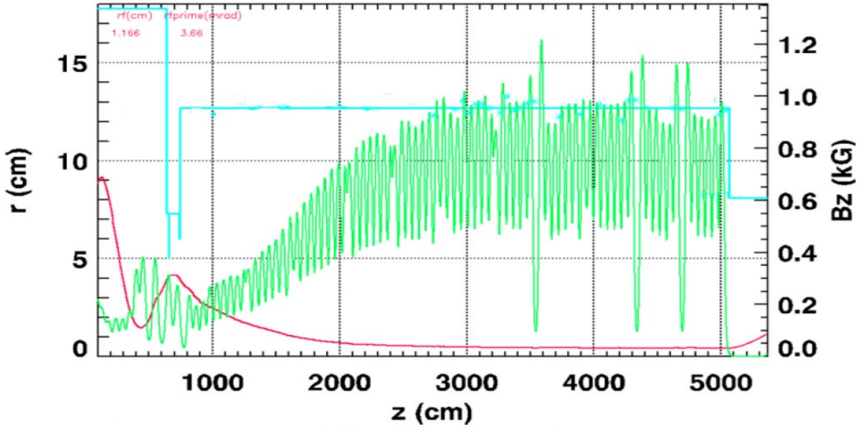
The solenoid scans shown in two previous slides have many distinct differences and similarities:

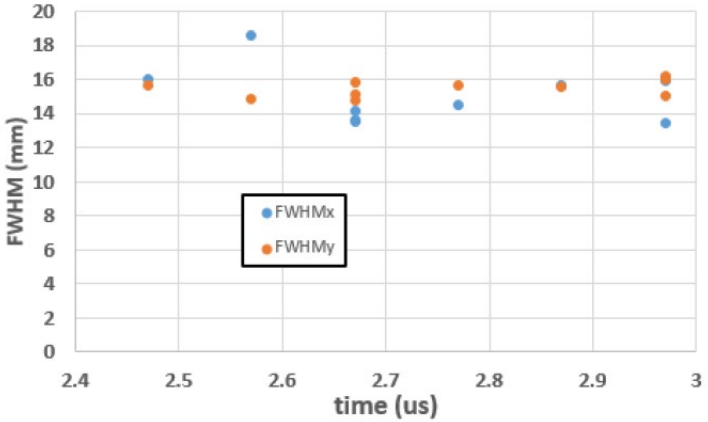
1. Both scans used the same solenoid magnet design.
2. The distance from the solenoid magnet to the target was over a factor of two higher for Station A.
3. The beam size in the solenoid was approximately factor of two higher for Station C.

A review of recent measurements suggest very little emittance growth in the DARHT II accelerator

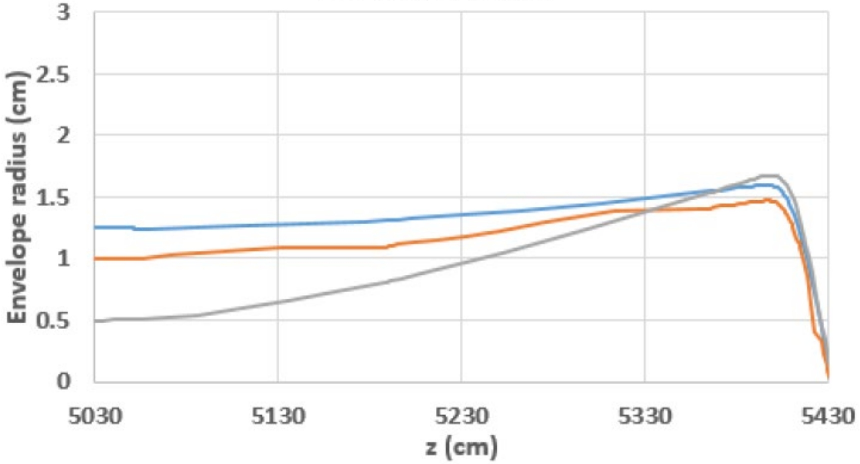
Recent (2017) Station A measurements

- The beam envelope radius at Station A (z=5370 cm) is about 1.2 cm. The normalized beam emittance in the LIA model is 210 π(mm-mrad).

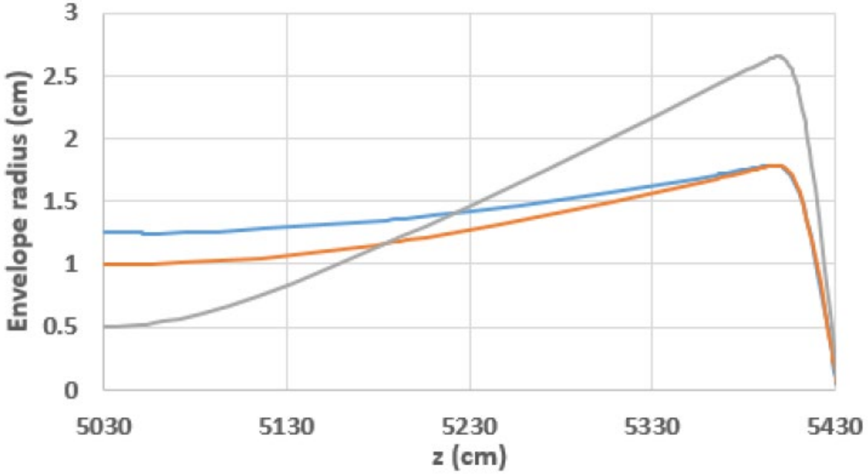




500 π(mm-mrad)



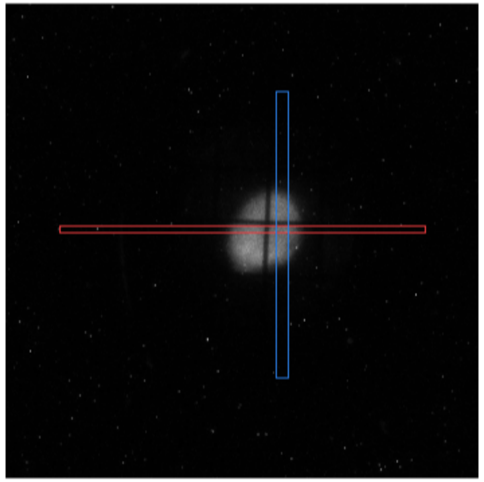
1000 π(mm-mrad)

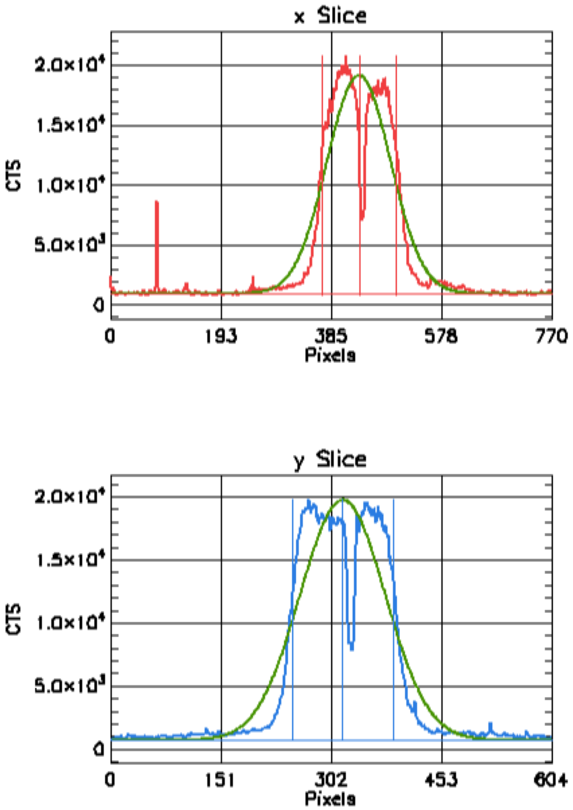


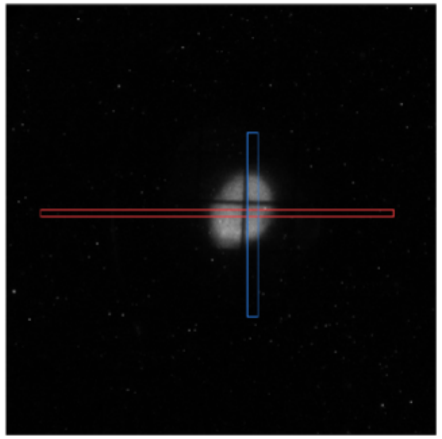
The small beam radius at the accelerator exit (0.5 cm) appears inconsistent with large emittance

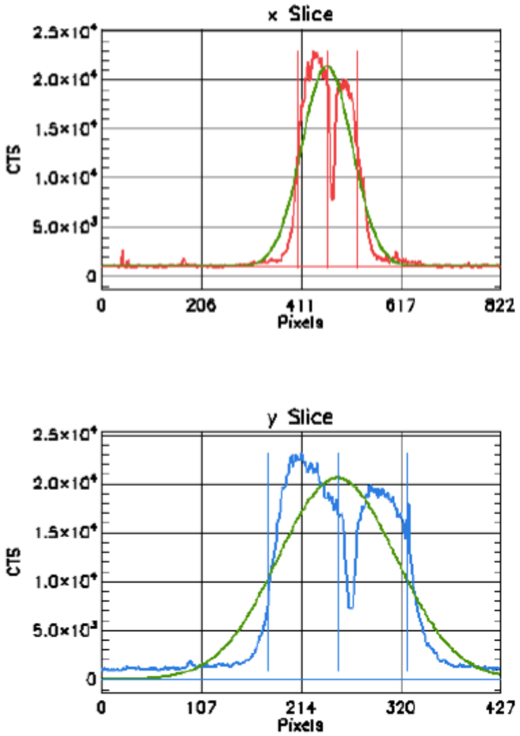
Station A beam profile measurements (LA-UR-18-26989

- Measurements at two times in the long pulse show very uniform profiles with little tails also suggesting very little emittance growth in the LIA


32783-2-TRSS1-shot-171114143215.tif

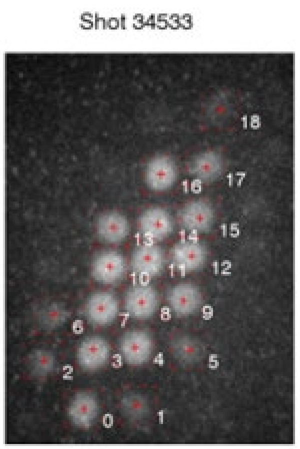


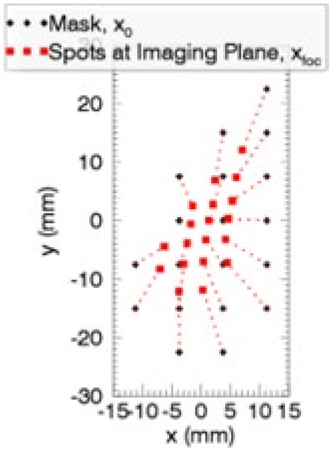

32784-2-TRSS1-shot-171114144227.tif

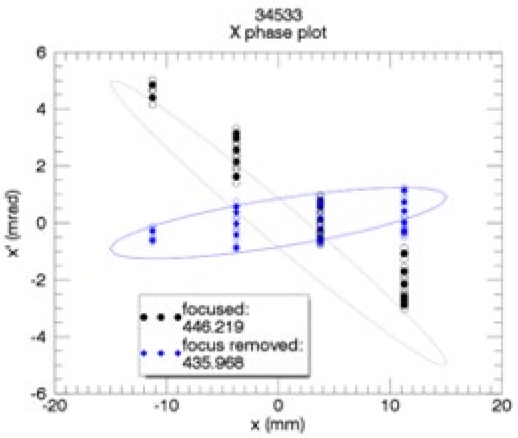


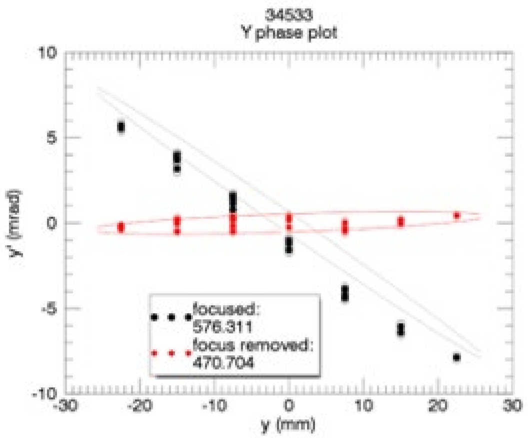
Preliminary results from recent (2019) Station C pepperpot measurements also suggest little emittance growth

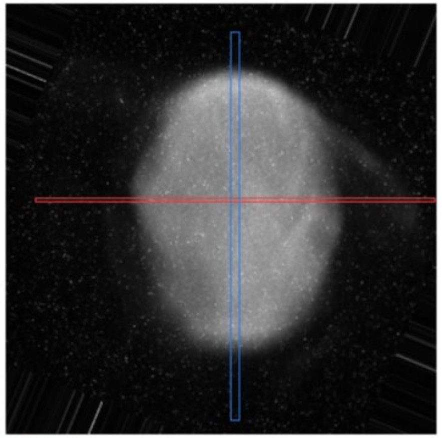
- The estimated normalized emittance is about 450 π(mm-mrad) and the beam profile is very uniform











35128-Frame-1.tif